UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN
FACULTAD DE INGENIERÍA MECÁNICA Y ELÉCTRICA
TIPO DE EXAMEN Y/O EVALUACIÓN:
FINAL ORDINARIO (*Final Exam*)

MATERIA/UNIDAD DE APRENDIZAJE: Temas Selectos de Optimización

LEARNING UNIT: *Selected Topics on Optimization* (in English)
SEMESTER: January – June 2024 (Spring)
ACADEMY: *Statistics and Operations Research* (Estadística e Investigación de Operaciones).
INSTRUCTOR: Dr. Roger Z. Ríos Mercado (ID 090969)

DIRECTIONS.- Answer the following questions and/or exercises in the answer sheet. Do not write in this sheet

SECTION 1: QUESTIONS (50 POINTS)

Answer and justify your answer.

1. [*UT2: Heuristics*; 5 pts] Explain when it is appropriate to use heuristics for solving a combinatorial optimization problem.
2. [*UT1: Combinatorial optimization*; 5 pts] When do we say a combinatorial optimization problem is “hard” to solve? Elaborate.
3. [*UT1: Combinatorial optimization*; 5 pts] When do we say a combinatorial optimization problem is “hard” to solve? Elaborate.
4. [*UT1: Combinatorial optimization*; 5 pts] Are there “easy” combinatorial optimization problems? Justify your answer.
5. [*UT2: Constructive heuristics;*5 pts] Explain clearly what a construction heuristic is for a combinatorial optimization problem.
6. [*UT2: Constructive heuristics;*5 pts] Do constructive heuristics guarantee to find a feasible solution to a given combinatorial optimization problem? Justify your answer.
7. [*UT2: Local search heuristics;* 5 pts] Explain clearly what a local search heuristic is for a combinatorial optimization problem.
8. [*UT2: Local search heuristics*; 5 pts] Explain the differences between the “Best Found” and “First Found” strategies within local search.
9. [*UT2: Local search heuristics;* 5 pts] Describe in detail the 2-OPT heuristic for the Traveling Salesman Problem. You may additionally illustrate your idea with an example or drawing.

10. [*UT2: Heuristics;* 5 pts] Explain in detail how you would compare two different heuristics for a given combinatorial optimization problem, that is, how you would determine which one is better.

SECTION 2: PROBLEMS (60 POINTS)

11. The Clustering Problem (CP) is defined as follows. Given a set n objects $V = \{1, 2, \ldots, n\}$, a number of clusters p, and a dissimilarity or "distance" measure d_{ij} between each pair of objects $i, j \in V$, which can also be represented by a matrix D = (d_{ij}), we must find a p-partition or p clusters (subsets of V) such that the objects within each cluster are the most similar possible according to this dissimilarity measure. For instance, if a given feasible solution is given by $X = (X_1, X_2, \ldots, X_p)$, where each X_k represents a subset of objects from V, then the objective function, or dissimilarity, is given by $f(X) = \sum_{k=1}^{p} d(X_k)$, where the dissimilarity of each subset is measured as $d(X_k) = \sum_{i,j \in X_k} d_{ij}$, that is, the sum of all the dissimilarity or distance values among all the elements in that subset. It is understood that if a subset is comprised of a single object, then its dissimilarity is zero. Recall that a p-partition $X = (X_1, X_2, \ldots, X_p)$ is valid or feasible if and only if the union of all subsets is equal to V and each element of V belongs to only one subset X_k. As an example, Figure 1 shows a dissimilarity or distance matrix for a set of 10 objects. Figure 2 shows how the dissimilarity function is computed for a given subset.

    The following questions refer to the CP instance depicted in Figure 1, assuming $p = 3$.

    (a) [*UT1: Combinatorial optimization;* 5 pts] Is $X^{(1)} = (\{3,6,8\}, \{1,4,9\}, \{2,5,10\})$ a feasible solution? Justify your answer.

    (b) [*UT1: Combinatorial optimization;* 5 pts] Is $X^{(2)} = (\{2,4,6,8,10\}, \{1,5,9\}, \{3,7\})$ a feasible solution? Justify your answer.

    (c) [*UT1: Combinatorial optimization;* 8 pts] Among the following three solutions, sort them from best to worst. Justify your answer.
    $X^{(3)} = (\{1,5,9\}, \{2,4,6,8,10\}, \{3,7\})$,
    $X^{(4)} = (\{2,5,9\}, \{1,3,4,8\}, \{6,10\})$,
    $X^{(5)} = (\{1,3,7,8\}, \{2,5,10\}, \{4,6,9\})$.

    (d) [*UT2: Constructive heuristics;* 12 pts] Starting from scratch, design a constructive heuristic for finding a feasible solution to the CP with n objects and p clusters. Show very clearly and precisely each step of your heuristic, either in pseudocode or a flow chart.

    (e) [*UT2: Constructive heuristics;* 6 pts] Illustrate how your heuristic works by applying it step by step in the example (Figure 1) to build a feasible solution to the problem. Was this solution better than solution $X^{(3)}$ from (c)?

    (f) [*UT2: Local search heuristics;* 8 pts] Given a feasible solution to the CP, design a local search heuristic for the problem. It suffices to describe **very clearly** how you define your move/neighborhood.

    (g) [*UT2: Local search heuristics;* 6 pts] Illustrate how your local search works starting from the following feasible solution $X^{(3)} = (\{2,4,6,8,10\}, \{1,5,9\}, \{3,7\})$. You may do just one **complete** iteration using the Best Found strategy. Did the solution improve?

$D =$

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}
v_1	0	12	26	14	31	8	46	35	20	10
v_2		0	41	9	19	33	28	39	14	30
v_3			0	25	9	15	37	20	47	23
v_4				0	29	12	34	8	15	18
v_5					0	45	31	16	11	27
v_6						0	13	29	40	32
v_7							0	36	9	21
v_8								0	17	46
v_9									0	23
v_{10}										0

Figure 1: Example matrix *D*. Only the upper-diagonal part is shown because *D* is symmetric.

$D =$

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}
v_1	0	12	26	14	31	8	46	35	20	10
v_2		0	41	9	19	33	28	39	14	30
v_3			0	25	9	15	37	20	47	23
v_4				0	29	12	34	8	15	18
v_5					0	45	31	16	11	27
v_6						0	13	29	40	32
v_7							0	36	9	21
v_8								0	17	46
v_9									0	23
v_{10}										0

Figure 2: Computation of the dissimilarity function for subset $X_1 = \{v_3, v_6, v_8\}$ is shown in the boxed cells. $d(X_1) = d_{36} + d_{38} + d_{68} = 15 + 20 + 29$.